c) $\forall n \in \mathbb{N}$; $V_n = V_0 \cdot q^n$

où $q = (2-\sqrt{3})^2$

et $V_0 = \dfrac{U_0 - \sqrt{3}}{U_0 + \sqrt{3}} = \dfrac{(-1-\sqrt{3})}{(-1+\sqrt{3})} \times \dfrac{(-1-\sqrt{3})}{(-1-\sqrt{3})}$

$= \dfrac{(1+\sqrt{3})^2}{-2}$

$= -\dfrac{1}{2}(1+\sqrt{3})^2$

Donc $\boxed{V_n = -\dfrac{1}{2}(1+\sqrt{3})^2 \cdot (2-\sqrt{3})^{2n} \quad \forall n \in \mathbb{N}}$

c) $\lim\limits_{n\to+\infty} V_n = \lim\limits_{n\to+\infty} -\dfrac{1}{2}(1+\sqrt{3})^2 (2-\sqrt{3})^{2n}$ ($(2-\sqrt{3})^{2n} \to 0$)

car $(2-\sqrt{3})^2 \simeq 0{,}07 \in]-1,1[$

Alors $\boxed{\lim\limits_{n\to+\infty} V_n = 0}$

d) $\forall n \in \mathbb{N}$; $V_n = \dfrac{U_n - \sqrt{3}}{U_n + \sqrt{3}}$

$\Rightarrow V_n(U_n + \sqrt{3}) = U_n - \sqrt{3}$

$\Rightarrow U_n(1 - V_n) = \sqrt{3}\,V_n + \sqrt{3}$